

XXVII. *On the Expansion by Heat of Metals and Alloys.* By A. MATTHIESSEN, F.R.S.

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IN a memoir “On the Expansion by Heat of Water and Mercury”*, I described a method of determining the expansion of bodies by weighing them in water at different temperatures. This method was chosen on account of its yielding accurate results with comparatively small quantities, for to purify large quantities of metal would entail immense labour and expense; so much so, in fact, that to purify sufficient quantities to make bars for the determination of the linear expansion would be practically impossible.

On determining the expansion of the metals by this method, I found that they did not expand regularly between 0° and 100° . The difference, however, between the rates of expansion between 0° and 50° and 50° and 100° was found to be so great that part of it might be possibly attributed to errors in the determination of the coefficients of expansion of water, as the coefficients of expansion of the metals are comparatively small when compared with those of water. Now, although the coefficient of expansion of mercury when determined by this method agrees with REGNAULT’S value, yet on account of this value being large, small errors in the water coefficients will not materially influence it; for the volume of water at

$$4^{\circ}=1\cdot0000, \text{ at } 100^{\circ}=1\cdot04316,$$

that of mercury at

$$0^{\circ}=1\cdot0000, \text{ at } 100^{\circ}=1\cdot01815,$$

and that of copper, for instance, at

$$0^{\circ}=1\cdot00000, \text{ at } 100^{\circ}=1004998,$$

showing that the expansion of copper is very small compared with that of water or mercury.

This fact led me to make another series of check experiments by determining the linear expansion of a certain copper bar, as described in the first part of my paper above quoted, and then weighing a piece of it, turned to the shape of a double cone, in water at different temperatures.

The following are the results obtained with the copper bar; and it may here be mentioned that copper does not behave in one respect like glass. The glass rods, as there shown, do not return directly to their original length after being heated to 100° and cooled rapidly; copper, however, does so; for no differences in the coefficients were observed after heating the rod to 100° , determining its expansion, allowing it to stand over night, and redetermining the coefficients.

* Philosophical Transactions, 1866, Part I.

In Table I., T , T_1 , T_2 , T_3 , T_4 indicate the temperatures in the order in which the observations were made; a , b , c the increment in length in millims. of the rod between T_1 and T_2 , T_1 and T_3 , T and T_3 respectively; the values in the three last columns are the coefficients of expansion of the rod between the observed temperatures. The length of the rod was 1804 millims. and its diameter about 15 millims.

TABLE I.

T_1 .	T_2 .	T_3 .	T_4 .	a .	b .	c .	$\frac{a}{T_2-T_1}$	$\frac{b}{T_3-T_1}$	$\frac{c}{T_3-T_4}$
7.4	99.9	7.0	2.807	2.803	0.03034	0.03017
7.0	50.7	100.1	10.4	1.263	2.808	2.718	0.02890	0.03016	0.03030
8.4	51.2	100.3	7.0	1.252	2.807	2.830	0.02925	0.03054	0.03033
7.5	51.9	100.3	7.8	1.295	2.805	2.803	0.02917	0.03023	0.03030
8.6	50.9	100.6	7.2	1.238	2.775	2.812	0.02927	0.03016	0.03011
Mean					0.02915	0.03026		

Or the mean coefficient between 0° and 50° may be taken $=0.02915$, and that between 0° and $100^\circ=0.03026$.

And taking the length of the rod at $0^\circ=1804$, it will be

$$\text{at } 50^\circ=1805.4575,$$

$$\text{at } 100^\circ=1808.0260.$$

From these values the linear expansion of the copper rod can be expressed by the formula

$$L_t=1804(1+0.00001555t+0.0000000122t^2),$$

or for any length of this sort of copper the formula for the correction of the linear expansion for temperature will be

$$L_t=L_0(1+0.00001555t+0.0000000122t^2),$$

and that for the correction of the cubical expansion

$$V_t=V_0(1+0.00004665t+0.0000000366t^2).$$

Two series of weighings in water were made with the piece cut from the end of the copper rod; the results are given in Table II.

TABLE II.

No. 1.

	Loss of weight in water =W.	$W(1+at)$.
11.0	3.95740	3.95885
50.7	3.91825	3.96640
97.1	3.81940	3.97590

No. 2.

10.6	3.95720	3.95850
54.2	3.91220	3.96685
97.3	3.81865	3.97570

The copper was slightly gilded to prevent the action of the water on it; the water being reboiled before each weighing to drive out any absorbed air. The values given in the third column express the volume of the copper in cubic centimetres, and are deduced from the observed loss of weight in water W , as described (p. 245) in the paper already quoted. Calculating formulæ to express the volumes of copper at different temperatures, we find for

No. 1.

$$V_t = 3.95680(1 + 0.00004645t + 0.0000000336t^2), \text{ or if } V_a = 1, \text{ then } V_{100} = 1.004981;$$

No. 2.

$V_t = 3.95655(1 + 0.00004681t + 0.0000000300t^2)$, or if $V_0 = 1$, then $V_{100} = 1.004981$, formulæ agreeing closely with that deduced from the determination of the linear expansion of the copper rod, namely,

$$V_t = V_0(1 + 0.00004665t + 0.0000000366t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005031.$$

This memoir may be divided into two parts:—

- I. *On the Expansion by Heat of the Metals.*
- II. *On the Expansion by Heat of Alloys.*

I. *On the Expansion by Heat of the Metals.*

The metals employed for these experiments were purified in the manner described in a former paper*, and cast in a well-smoked mould, which gave the casting the shape of a double wedge, as shown in fig. 7†. Owing to the action of the water on some of the metals and alloys, the castings had in some cases to be varnished or gilded, the latter method being more generally used. To prove that the gilding or varnishing had no influence on the results, some of those metals on which water has no action were varnished or gilded in the one series and not in the other (Series Nos. 3 and 4, 14 and 15, 11 and 12).

The disposition of the apparatus and the method of observation was the same as described in the paper “On the Expansion of Water and Mercury.”

Sometimes observations were made commencing at the highest temperature, and cooling down without boiling out the water between the determinations made at the different temperatures. (The series made in this manner will be headed once boiled.)

At others the observations were first taken at the lowest temperature, the water reboiled, then those at the highest temperature, and afterwards on cooling those at the intermediate one (twice boiled).

And again, at others the water was reboiled between each set of observations (thrice boiled).

It may be as well to point out some of the causes of failure in this method of deter-

* Philosophical Transactions, 1860, p. 177.

† Philosophical Transactions, 1866, Plate XX.

mining the coefficients of expansion; for although it appears a very simple one, yet it requires, I may say, very great care to ensure reliable results.

I. Very often a series was rendered valueless by small hairs or particles of dust falling into the water and attaching themselves to the fine platinum wire, as 0.1 or 0.2 milligramme difference in the weight makes a considerable difference in the expansion of those metals or alloys which have a low coefficient of expansion; this source of error was carefully guarded against; in fact, if on taking the cylinder out to reboil the water any particle of dust floated near the wires, the observations were considered worthless, and fresh ones taken after reboiling the water.

II. When, for instance, the observations were being made at the highest temperature, if by chance the temperature sunk three or four degrees and gradually rose again, the air absorbed at the lower temperature would be expelled, and an air-bubble would sometimes attach itself to the metal and cause a false weighing.

III. If the casting were not perfect, or were crystalline, and the air could not be completely expelled from the small cavities by boiling, results were obtained which did not agree together, owing to its expansion at the high and partial absorption by the water at the lower temperature. For this reason two series were always made with each metal and alloy, and where possible they were recast.

The values obtained when weighing the purified metals in water were as follow:—

TABLE III.—Cadmium.

Owing to this metal becoming crystalline at about 80°, four observations were made between 0° and 100°. It will be seen that this change of molecular condition has no effect on this physical property.

No. 3.—Cadmium, four times boiled.

T.	Loss of weight in water = W.	$W(1+at)$.	Calculated.	Difference.
8.0	6.29175	6.29250	6.29241	+ 0.00009
45.4	6.25080	6.31280	6.31309	- 0.00029
77.4	6.16450	6.33255	6.33249	+ 0.00015
95.2	6.10230	6.34380	6.34377	+ 0.00003

$$V_t = 6.28826(1 + 10^{-4}0.7991t + 10^{-6} \times 0.163t^2)^*, \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.009611.$$

No. 4.—Cadmium varnished, four times boiled.

8.1	5.91135	5.91210	5.91182	+ 0.00028
47.9	5.86800	5.93265	5.93274	- 0.00009
75.2	5.79895	5.94885	5.94885	0.00000
93.8	5.73915	5.96040	5.96065	- 0.00025

$$V_t = 5.90793(1 + 10^{-4}0.7991t + 10^{-6} \times 0.163t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.009611.$$

* I have employed this method of writing the formulæ to prevent mistakes in the number of the zeros, as well as to show at a glance their number. I have also preferred keeping the exponents constant, adding, instead of a tering them, a zero after the decimal point where required.

TABLE III. (continued.)

No. 5.—Zinc varnished, thrice boiled.

T.	Loss of weight in water = \bar{W} .	$W(1+at)$.
10·0	7·42245	7·42445
50·6	7·35985	7·45000
95·7	7·19460	7·48200

$$V_t = 7·41863(1 + 10^{-4} \times 0·7719t + 10^{-6} \times 0·126t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·008979.$$

No. 6.—Zinc varnished, thrice boiled.

9·0	6·69705	6·69830
50·6	6·64010	6·72285
95·0	6·49345	6·74945

$$V_t = 6·69304(1 + 10^{-4} \times 0·8726t + 10^{-6} \times 0·0153t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·008879.$$

No. 7.—Lead varnished, once boiled.

14·0	4·55540	4·55880
51·2	4·51635	4·57295
94·1	4·41840	4·58960

$$V_t = 4·55355(1 + 10^{-4} \times 0·8215t + 10^{-6} \times 0·0210t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·008425.$$

No. 8.—Lead varnished, once boiled.

14·7	4·40320	4·40695
52·9	4·36270	4·42090
90·7	4·27960	4·43500

$$V_t = 4·40166(1 + 10^{-4} \times 0·8140t + 10^{-6} \times 0·0234t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·008374.$$

No. 9.—Tin, once boiled.

10·2	3·66860	3·66965
55·0	3·62850	3·68060
93·2	3·55520	3·69070

$$V_t = 3·66730(1 + 10^{-4} \times 0·6237t + 10^{-6} \times 0·0656t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·006893.$$

No. 10.—Tin, once boiled.

8·2	4·28625	4·28680
48·5	4·25005	4·29800
94·8	4·14945	4·31250

$$V_t = 4·28468(1 + 10^{-4} \times 0·5964t + 10^{-6} \times 0·0922t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·006886.$$

No. 11.—Silver gilded, thrice boiled.

10·7	5·33570	5·33750
52·7	5·28040	5·35035
94·3	5·16280	5·36370

$$V_t = 5·33433(1 + 10^{-4} \times 0·5523t + 10^{-6} \times 0·0335t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·005858.$$

TABLE III. (continued.)

No. 12.—Silver, thrice boiled.

T.	Loss of weight in water= \bar{W} .	$W(1+at)$.
7.5	5.49925	5.49975
50.5	5.44655	5.51300
97.1	5.37080	5.52845

$$V_t = 5.49754(1 + 10^{-4} \times 0.5330t + 10^{-6} \times 0.0475t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005805.$$

No. 13.—Copper gilded, twice boiled.

10.2	5.82730	5.82900
59.8	5.74595	5.84280
95.1	5.63155	5.85400

$$V_t = 5.82644(1 + 10^{-4} \times 0.4223t + 10^{-6} \times 0.0791t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005014.$$

No. 14.—Gold varnished, thrice boiled.

9.1	2.44605	2.44655
45.4	2.42625	2.45030
95.1	2.36250	2.45585

$$V_t = 2.44565(1 + 10^{-4} \times 0.4008t + 10^{-6} \times 0.0397t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.004405.$$

No. 15.—Gold, once boiled.

10.7	2.44415	2.44490
47.8	2.42225	2.44880
93.0	2.36405	2.45380

$$V_t = 2.44381(1 + 10^{-4} \times 0.4142t + 10^{-6} \times 0.0276t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.004418.$$

No. 16.—Bismuth varnished, once boiled.

8.9	4.29915	4.29995
51.2	4.25350	4.30680
96.0	4.14795	4.31455

$$V_t = 4.29857(1 + 10^{-4} \times 0.3591t + 10^{-6} \times 0.0294t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.003885.$$

No. 17.—Bismuth varnished, twice boiled.

11.6	5.16400	5.16625
50.0	5.11255	5.17375
93.5	4.99200	5.18335

$$V_t = 5.16416(1 + 10^{-4} \times 0.3413t + 10^{-6} \times 0.0599t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.004012.$$

No. 18.—Palladium (purified metal, lent by Messrs. JOHNSON and MATTHEY), thrice boiled.

7.5	7.48500	7.48700
55.5	7.38975	7.49785
97.5	7.21130	7.50875

$$V_t = 7.48545(1 + 10^{-4} \times 0.2708t + 10^{-6} \times 0.0497t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.003205.$$

TABLE III. (continued.)

No. 19.—Palladium, same piece as used for the last series, repolished, once boiled.

T.	Loss of weight in water= \bar{W} .	$W(1+at)$.
9.8	7.48420	7.48610
56.0	7.38795	7.49785
97.4	7.21140	7.50855

$$V_t = 748363(1 + 10^{-4} \times 0.3357t + 10^{-6} \times 0.0629t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.003420.$$

No. 20.—Antimony gilded, once boiled.

11.7	6.26320	6.06590
57.2	5.98155	6.07420
97.1	5.84300	6.08245

$$V_t = 6.06396(1 + 10^{-4} \times 0.2686t + 10^{-6} \times 0.0469t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.003155.$$

No. 21.—Antimony gilded, once boiled.

10.9	6.20525	6.20745
61.0	6.11010	6.21705
97.8	5.97670	6.22475

$$V_t = 6.20550(1 + 10^{-4} \times 0.2854t + 10^{-6} \times 0.0325t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.003179.$$

No. 22.—Platinum (purified metal, lent by Messrs. JOHNSON and MATHEY), twice boiled.

8.5	2.22465	2.22500
56.0	2.19510	2.22775
97.3	2.14215	2.23025

$$V_t = 2.22452(1 + 10^{-4} \times 0.2516t + 10^{-6} \times 0.0134t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.002650.$$

No. 23.—Platinum, same piece as used for the last series, repolished, twice boiled.

6.6	2.22430	2.22440
51.8	2.19890	2.22705
96.7	2.14260	2.22975

$$V_t = 2.22402(1 + 10^{-4} \times 0.2591t + 10^{-6} \times 0.0074t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.002665.$$

The means of the foregoing formulæ are put together in Table IV.

TABLE IV.

Formulæ for correcting the cubical expansion.

Cadmium	$V_t = V_0(1 + 10^{-4} \times 0.8078t + 10^{-6} \times 0.140t^2)$
Zinc	$V_t = V_0(1 + 10^{-4} \times 0.8222t + 10^{-6} \times 0.0706t^2)$
Lead	$V_t = V_0(1 + 10^{-4} \times 0.8177t + 10^{-6} \times 0.0222t^2)$
Tin	$V_t = V_0(1 + 10^{-4} \times 0.6100t + 10^{-6} \times 0.0789t^2)$
Silver	$V_t = V_0(1 + 10^{-4} \times 2.5426t + 10^{-6} \times 0.0405t^2)$
Copper*	$V_t = V_0(1 + 10^{-4} \times 0.4463t + 10^{-6} \times 0.0555t^2)$
Gold	$V_t = V_0(1 + 10^{-4} \times 0.4075t + 10^{-6} \times 0.0336t^2)$
Bismuth	$V_t = V_0(1 + 10^{-4} \times 0.3502t + 10^{-6} \times 0.0446t^2)$
Palladium	$V_t = V_0(1 + 10^{-4} \times 0.3032t + 10^{-6} \times 0.0280t^2)$
Antimony	$V_t = V_0(1 + 10^{-4} \times 0.2770t + 10^{-6} \times 0.0397t^2)$
Platinum	$V_t = V_0(1 + 10^{-4} \times 0.2554t + 10^{-6} \times 0.0104t^2)$

The formulæ for the correction of the linear expansion of the above metals may be deduced by dividing the coefficients obtained for the linear expansion by 3. These values are given in Table V.

TABLE V.

Formulæ for correcting the linear expansion.

Metal.	
Cadmium	$L_t = L_0(1 + 10^{-4} \times 0.2693t + 10^{-6} \times 0.0466t^2)$
Zinc	$L_t = L_0(1 + 10^{-4} \times 0.2741t + 10^{-6} \times 0.0234t^2)$
Lead	$L_t = L_0(1 + 10^{-4} \times 0.2726t + 10^{-6} \times 0.0074t^2)$
Tin	$L_t = L_0(1 + 10^{-4} \times 0.2033t + 10^{-6} \times 0.0263t^2)$
Silver	$L_t = L_0(1 + 10^{-4} \times 0.1809t + 10^{-6} \times 0.0135t^2)$
Copper	$L_t = L_0(1 + 10^{-4} \times 0.1481t + 10^{-6} \times 0.0185t^2)$
Gold	$L_t = L_0(1 + 10^{-4} \times 0.1358t + 10^{-6} \times 0.0112t^2)$
Bismuth	$L_t = L_0(1 + 10^{-4} \times 0.1167t + 10^{-6} \times 0.0149t^2)$
Palladium	$L_t = L_0(1 + 10^{-4} \times 0.1011t + 10^{-6} \times 0.0093t^2)$
Antimony	$L_t = L_0(1 + 10^{-4} \times 0.0923t + 10^{-6} \times 0.0132t^2)$
Platinum	$L_t = L_0(1 + 10^{-4} \times 0.0851t + 10^{-6} \times 0.0035t^2)$

These values agree in most instances with those found by former observers; and as these only determined the expansion between 0° and 100°, I give in the following Table the volume and length which a unit volume or length at 0° will occupy at 100° as deduced from the formulæ.

TABLE VI.

	$V_0 = 1,$ then $V_{100} =$	$L_0 = 1,$ then $L_{100} =$
Cadmium	1.009478	1.003159
Zinc	1.008928	1.002976
Lead	1.008399	1.002799
Tin	1.006889	1.002296
Silver	1.005831	1.001943
Copper	1.004998	1.001866
Gold	1.004411	1.001470
Bismuth	1.003948	1.001316
Palladium	1.003312	1.001104
Antimony	1.003167	1.001056
Platinum	1.002658	1.000886

* The formula here given is the mean of Series No. 1 and 2, and No. 13.

On the Expansion by Heat of Alloys.

The alloys were made in the manner described in a former paper*, and the results obtained are contained in Table VII.

I have grouped them together in the same way as I did when speaking of their electric conducting-power.

TABLE VII.

No. 24.—Sn₄Pb gilded, thrice boiled.

T.	Loss of weight in water = W.	W(1+at).
12·0	7·04010	7·04345
51·5	6·97425	7·06265
95·0	6·81730	7·08610

$$V_t = 7·03801(1 + 10^{-4} \times 0·6331t + 10^{-6} \times 0·0907t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·007238.$$

No. 25.—Sn₄Pb gilded, once boiled.

15·0	7·03845	7·04475
59·4	6·95050	7·06620
95·1	6·81635	7·08560

$$V_t = 7·03817(1 + 10^{-4} \times 0·6068t + 10^{-6} \times 0·107t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·007138.$$

No. 26.—Pb₄Sn gilded, twice boiled.

11·0	5·26995	5·27190
54·8	5·21670	5·29115
95·8	5·10535	5·30965

$$V_t = 5·26715(1 + 10^{-4} \times 0·8171t + 10^{-6} \times 0·0263t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·008434.$$

No. 27.—Pb₄Sn gilded, once boiled.

11·0	5·27395	5·27590
52·8	5·22465	5·29410
95·6	5·10975	5·31350

$$V_t = 5·27124(1 + 10^{-4} \times 0·8004t + 10^{-6} \times 0·0401t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·008405.$$

No. 28.—Cd Pb gilded, twice boiled.

8·6	5·36850	5·36935
56·4	5·31245	5·39260
98·1	5·19620	5·41305

$$V_t = 5·36519(1 + 10^{-4} \times 0·9012t + 10^{-6} \times 0·0834t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·909095.$$

No. 29.—Cd Pb gilded, once boiled.

9·2	5·60060	5·60175
60·4	5·53285	5·62790
91·7	5·44265	5·64415

$$V_t = 5·59711(1 + 10^{-4} \times 0·8998t + 10^{-6} \times 0·018t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1·009180.$$

* Philosophical Transactions, 1860.

TABLE VII. (continued.)

No. 30.—Sn₄ Zn gilded, once boiled.

T.	Loss of weight in water= \bar{W} .	$W(1+at)$.
9.3	6.58530	6.58670
54.7	6.51495	6.60765
95.9	6.37195	6.62740

$$V_t = 6.58252(1 + 10^{-4} \times 0.6809t + 10^{-6} \times 0.0314t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.007123.$$

No. 31.—Sn₄ Zn gilded, once boiled.

10.7	6.58830	6.59055
60.9	6.49990	6.61330
94.5	6.38165	6.63095

$$V_t = 6.58626(1 + 10^{-4} \times 0.5945t + 10^{-6} \times 0.130t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.007245.$$

No. 32.—Sn₆ Zn gilded, once boiled.

11.1	6.85455	6.85715
57.9	6.77180	6.87915
96.8	6.62940	6.89960

$$V_t = 6.85235(1 + 10^{-4} \times 0.6205t + 10^{-6} \times 0.0948t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.007153.$$

No. 33.—Sn₆ Zn gilded, thrice boiled.

14.3	7.23135	7.23705
52.7	7.16090	7.25575
91.4	7.01775	7.27610

$$V_t = 7.23047(1 + 10^{-4} \times 0.6268t + 10^{-6} \times 0.0697t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.006965.$$

No. 34.—Bi₄₄ Sn gilded, thrice boiled.

16.2	5.49395	5.49995
53.2	5.43465	5.50795
91.1	5.32160	5.51635

$$V_t = 5.49651(1 + 10^{-4} \times 0.3842t + 10^{-6} \times 0.0132t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.003974.$$

No. 35.—Bi₄₄ Sn gilded, once boiled.

10.8	6.10665	6.10885
57.8	6.02515	6.12040
94.0	5.90165	6.13005

$$V_t = 6.10635(1 + 10^{-4} \times 0.3743t + 10^{-6} \times 0.0410t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.004153.$$

No. 36.—Bi Sn₂ gilded, once boiled.

10.2	6.04220	6.04395
58.6	5.96205	6.05875
96.8	5.83300	6.07065

$$V_t = 6.04087(1 + 10^{-4} \times 0.4987t + 10^{-6} \times 0.0110t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005097.$$

TABLE VII. (continued.)

No. 37.—Bi Sn₂ gilded, once boiled.

T.	Loss of weight in water=W.	W(1+at)
10.9	6.05175	6.05390
57.5	5.97470	6.06820
97.6	5.83920	6.08070

$$V_t = 6.05059(1 + 10^{-4} \times 0.5008t + 10^{-6} \times 0.00925t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005100.$$

No. 38.—Bi₂₄ Pb gilded, once boiled.

10.2	6.72710	6.72905
56.6	6.64040	6.74125
94.4	6.49825	6.75160

$$V_t = 6.72644(1 + 10^{-4} \times 0.3781t + 10^{-6} \times 0.0192t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.003973.$$

No. 39.—Bi₂₄ Pb gilded, once boiled.

13.2	6.11480	6.11865
50.4	6.05440	6.12800
90.9	5.92270	6.13865

$$V_t = 6.11543(1 + 10^{-4} \times 0.3955t + 10^{-6} \times 0.0245t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.004200.$$

No. 40.—Bi Pb₂ gilded, once boiled.

11.1	5.97110	5.97335
56.5	5.90675	5.99615
96.8	5.78095	6.01655

$$V_t = 5.96780(1 + 10^{-4} \times 0.8362t + 10^{-6} \times 0.00782t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.008440.$$

No. 41.—Bi Pb₂ gilded, once boiled.

10.6	6.17070	6.17275
53.1	6.11340	6.19555
95.0	5.98260	6.21850

$$V_t = 6.16714(1 + 10^{-4} \times 0.8561t + 10^{-6} \times 0.0216t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.008777.$$

No. 42.—Bi Pb₂, gilded, once boiled.

12.4	5.17255	5.17520
58.1	5.11390	5.19550
96.8	5.00855	5.21300

$$V_t = 5.16976(1 + 10^{-4} \times 0.8463 + 10^{-6} \times 0.0183t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.008646.$$

No. 43.—Alloy of copper and zinc, containing 71.0 per cent. by weight copper, determined by analysis, gilded, thrice boiled.

9.8	7.61120	7.61315
57.4	7.51595	7.63315
97.1	7.35020	7.65145

$$V_t = 7.60930(1 + 10^{-4} \times 0.5109t + 10^{-6} \times 0.0614t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005723.$$

TABLE VII. (continued.)

No. 44.—No. 43 repeated.

T.	Loss of weight in water= \bar{W} .	$W(1+at)$.	Calculated.
11.2	7.61090	7.61385	
58.9	7.51100	7.63405	
96.5	7.35315	7.65120	

$$V_t = 7.60936(1 + 10^{-4} \times 0.5213t + 10^{-6} \times 0.0503t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005716.$$

No. 45.—Au Sn₂, thrice boiled.

9.8	5.24965	5.25100
51.6	5.19420	5.26025
97.4	5.06210	5.27070

$$V_t = 5.2487(1 + 10^{-4} \times 0.4124t + 10^{-6} \times 0.0149t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.004273.$$

No. 46.—Au Sn₂, twice boiled.

10.5	5.44040	5.44215
55.7	5.37295	5.45210
97.1	5.24700	5.46205

$$V_t = 5.43997(1 + 10^{-4} \times 0.3763t + 10^{-6} \times 0.0429t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.004192.$$

No. 47.—Au₂ Sn, twice boiled.

12.0	5.57075	5.57340
55.1	5.50440	5.58380
96.8	5.37535	5.59445

$$V_t = 5.57061(1 + 10^{-4} \times 0.4132t + 10^{-6} \times 0.0298t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.004430.$$

No. 48.—Au₂ Sn₇, gilded, twice boiled.

12.1	5.58500	5.58775
56.3	5.51560	5.59850
95.3	5.39450	5.60840

$$V_t = 5.58489(1 + 10^{-4} \times 0.4198t + 10^{-6} \times 0.0229t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.004427.$$

No. 49.—Ag₄ Au, thrice boiled.

9.7	3.64395	3.64485	3.64485
51.4	3.60725	3.65270	3.65270
96.1	3.51950	3.66110	3.66110

$$V_t = 3.64303(1 + 10^{-4} \times 0.5163t), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005163.$$

No. 50.—Ag₄ Au, thrice boiled.

10.8	3.76475	3.76605	3.76604
58.5	3.71525	3.77530	3.77532
92.8	3.64415	3.78200	3.78200

$$V_t = 3.76394(1 + 10^{-4} \times 0.5170t), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005170.$$

TABLE VII. (continued.)

No. 51.—Ag Au, thrice boiled.

T.	Loss of weight in water=W.	$W(1+at)$.	Calculated.
8.6	3.99445	3.99510	3.99510
57.3	3.94340	4.00470	4.00465
98.4	3.85115	4.01270	4.01270

$V_t = 3.99342(1 + 10^{-4} \times 0.4906t)$, or if $V_0 = 1$, then $V_{100} = 1.004906$.

No. 52.—Ag Au, thrice boiled.

9.0	3.99005	3.99080	3.99080
58.3	3.93730	4.00055	4.00055
98.0	3.84800	4.00830	4.00831

$V_t = 3.98904(1 + 10^{-4} \times 0.4927t)$, or if $V_0 = 1$, then $V_{100} = 1.004927$.

No. 53.—Ag Au₄, thrice boiled.

9.6	2.90470	2.90540
60.6	2.86165	2.91110
94.1	2.80700	2.91585

$V_t = 2.90453(1 + 10^{-4} \times 0.2989t + 10^{-6} \times 0.123t^2)$, or if $V_0 = 1$, then $V_{100} = 1.004119$.

No. 54.—Ag Au₄, thrice boiled.

8.4	2.98805	2.98850
56.7	2.94930	2.99425
93.5	2.88895	2.99970

$V_t = 2.98766(1 + 10^{-4} \times 0.3241t + 10^{-6} \times 0.114t^2)$, or if $V_0 = 1$, then $V_{100} = 1.004381$.

No. 55.—Alloy of silver and platinum, containing 66.6 per cent. by weight silver (lent by Messrs. JOHNSON and MATTHEY), thrice boiled.

7.0	4.67260	4.67290
58.4	4.60940	4.68370
96.3	4.50995	4.69200

$V_t = 4.67147(1 + 10^{-4} \times 0.4359t + 10^{-6} \times 0.0213t^2)$, or if $V_0 = 1$, then $V_{100} = 1.004572$.

No. 56.—No. 55 repeated, thrice boiled.

9.4	4.67330	4.67435
59.4	4.60800	4.68470
96.8	4.50930	4.69310

$V_t = 4.67252(1 + 10^{-4} \times 0.4134t + 10^{-6} \times 0.0431t^2)$, or if $V_0 = 1$, then $V_{100} = 1.004565$.

No. 57.—Alloy of gold and copper, containing 66.6 per cent. gold, gilded, once boiled.

11.0	4.52820	4.52985
59.4	4.46525	4.53965
97.7	4.36740	4.54835

$V_t = 4.52781(1 + 10^{-4} \times 0.4029t + 10^{-6} \times 0.0629t^2)$, or if $V_0 = 1$, then $V_{100} = 1.004658$.

TABLE VII. (continued.)

No. 58.—No. 57 repeated, twice boiled.

T.	Loss of weight in water= \bar{W} .	$W(1+at)$.
10.0	4.52845	4.52965
54.2	4.47595	4.53850
96.9	4.36970	4.54815

$$V_t = 4.52781(1 + 10^{-4} \times 0.4002t + 10^{-6} \times 0.0655t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.004657.$$

No. 59.—Alloy of silver and copper, containing 36.1 per cent. by weight, silver determined by analysis, thrice boiled.

8.5	3.19535	3.19585
52.5	3.16165	3.20325
95.1	3.08880	3.21080

$$V_t = 3.19447(1 + 10^{-4} \times 0.5065t + 10^{-6} \times 0.327t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005392.$$

No. 60.—No. 59 repeated, thrice boiled.

9.6	3.19525	3.19600
56.4	3.15620	3.20380
96.0	3.08725	3.21125

$$V_t = 3.19454(1 + 10^{-4} \times 0.4704t + 10^{-6} \times 0.778t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005482.$$

No. 61.—Alloy of silver and copper, containing 71.6 per cent. by weight, silver determined by analysis, gilded, thrice boiled.

7.0	5.85330	5.85370
53.1	5.78990	5.86770
97.4	5.65130	5.88415

$$V_t = 5.85185(1 + 10^{-4} \times 0.4421t + 10^{-6} \times 0.128t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.00570.$$

No. 62.—No. 61 repeated, regilded, once boiled.

10.1	5.85335	5.85500
52.4	5.79205	5.86795
98.1	5.64930	5.88505

$$V_t = 5.85232(1 + 10^{-4} \times 0.4406t + 10^{-6} \times 0.132t^2), \text{ or if } V_0 = 1, \text{ then } V_{100} = 1.005726.$$

The alloys No. 24 to 33 belong to the first group, namely, those made of the metals which, when alloyed with one another, conduct electricity in the ratio of their relative volumes; from 34 to 48 to the third group, namely, those made of the metals which when alloyed with one another, or with one of those belonging to the first group of metals and alloys (these form the second group No. 49 to 62), conduct electricity in a lower degree than that calculated from the mean of their volumes.

In Table VIII. the mean formulæ are given for the correction of expansion by heat for the foregoing alloys; in Table IX. those for the correction of the linear expansion by heat.

TABLE VIII.

Alloy.	Mean correction formulæ for cubical expansion by heat.
Sn ₄ Pb	$V_t = V_0(1 + 10^{-4} \times 0.6200t + 10^{-6} \times 0.0988t^2)$
Pb ₄ Sn	$V_t = V_0(1 + 10^{-4} \times 0.8087t + 10^{-6} \times 0.0332t^2)$
Cd Pb	$V_t = V_0(1 + 10^{-4} \times 0.9005t + 10^{-6} \times 0.0133t^2)$
Sn ₄ Zn	$V_t = V_0(1 + 10^{-4} \times 0.6377t + 10^{-6} \times 0.0807t^2)$
Sn ₆ Zn	$V_t = V_0(1 + 10^{-4} \times 0.6236t + 10^{-6} \times 0.0822t^2)$
Bi ₄₄ Sn	$V_t = V_0(1 + 10^{-4} \times 0.3793t + 10^{-6} \times 0.0271t^2)$
Bi Sn ₂	$V_t = V_0(1 + 10^{-4} \times 0.4997t + 10^{-6} \times 0.0101t^2)$
Bi ₂₄ Pb	$V_t = V_0(1 + 10^{-4} \times 0.3868t + 10^{-6} \times 0.0218t^2)$
Bi Pb ₂	$V_t = V_0(1 + 10^{-4} \times 0.8462t + 10^{-6} \times 0.0159t^2)$
Cu + Zn (71 per cent. Cu)	$V_t = V_0(1 + 10^{-4} \times 0.5161t + 10^{-6} \times 0.0558t^2)$
Au Sn ₂	$V_t = V_0(1 + 10^{-4} \times 0.3944t + 10^{-6} \times 0.0289t^2)$
Au ₂ Sn ₇	$V_t = V_0(1 + 10^{-4} \times 0.4165t + 10^{-6} \times 0.0263t^2)$
Ag ₄ Au	$V_t = V_0(1 + 10^{-4} \times 0.5166t)$
Ag Au	$V_t = V_0(1 + 10^{-4} \times 0.4916t)$
Ag Au ₄	$V_t = V_0(1 + 10^{-4} \times 0.3115t + 10^{-6} \times 0.1185t^2)$
Ag + Pt (66.6 per cent. Ag)	$V_t = V_0(1 + 10^{-4} \times 0.4246t + 10^{-6} \times 0.0322t^2)$
Au + Cu (66.6 per cent. Au)	$V_t = V_0(1 + 10^{-4} \times 0.4015t + 10^{-6} \times 0.0642t^2)$
Ag + Cu (36.1 per cent. Ag)	$V_t = V_0(1 + 10^{-4} \times 0.4884t + 10^{-6} \times 0.0552t^2)$
Ag + Cu (71.6 per cent. Ag)	$V_t = V_0(1 + 10^{-4} \times 0.4413t + 10^{-6} \times 0.130t^2)$

TABLE IX.

Alloy.	Correction formulæ for linear expansion by heat.
Sn ₄ Pb	$L_t = L_0(1 + 10^{-4} \times 0.2066t + 10^{-6} \times 0.0329t^2)$
Pb ₄ Sn	$L_t = L_0(1 + 10^{-4} \times 0.2696t + 10^{-6} \times 0.0111t^2)$
Cd Pb	$L_t = L_0(1 + 10^{-4} \times 0.3002t + 10^{-6} \times 0.0044t^2)$
Sn ₄ Zn	$L_t = L_0(1 + 10^{-4} \times 0.2126t + 10^{-6} \times 0.0269t^2)$
Sn ₆ Zn	$L_t = L_0(1 + 10^{-4} \times 0.2079t + 10^{-6} \times 0.0274t^2)$
Bi ₄₁ Sn	$L_t = L_0(1 + 10^{-4} \times 0.1264t + 10^{-6} \times 0.0090t^2)$
Bi Sn ₂	$L_t = L_0(1 + 10^{-4} \times 0.1666t + 10^{-6} \times 0.0034t^2)$
Bi ₂₄ Pb	$L_t = L_0(1 + 10^{-4} \times 0.1293t + 10^{-6} \times 0.0073t^2)$
Bi Pb ₂	$L_t = L_0(1 + 10^{-4} \times 0.2821t + 10^{-6} \times 0.0053t^2)$
Cu + Zn (71 per cent. Cu)	$L_t = L_0(1 + 10^{-4} \times 0.1720t + 10^{-6} \times 0.0186t^2)$
Au Sn ₂	$L_t = L_0(1 + 10^{-4} \times 0.1315t + 10^{-6} \times 0.0096t^2)$
Au ₂ Sn	$L_t = L_0(1 + 10^{-4} \times 0.1388t + 10^{-6} \times 0.0088t^2)$
Ag Au	$L_t = L_0(1 + 10^{-4} \times 0.1722t)$
Ag Au ₄	$L_t = L_0(1 + 10^{-4} \times 0.1638t)$
Ag Au ₁	$L_t = L_0(1 + 10^{-4} \times 0.1038t + 10^{-6} \times 0.0395t^2)$
Ag + Pt (66.6 per cent. Pt)	$L_t = L_0(1 + 10^{-4} \times 0.1415t + 10^{-6} \times 0.0107t^2)$
Au + Cu (66.6 per cent. Au)	$L_t = L_0(1 + 10^{-4} \times 0.1338t + 10^{-6} \times 0.0214t^2)$
Ag + Cu (31.6 per cent. Ag)	$L_t = L_0(1 + 10^{-4} \times 0.1628t + 10^{-6} \times 0.0182t^2)$
Ag + Cu (76.1 per cent. Ag)	$L_t = L_0(1 + 10^{-4} \times 0.1471t + 10^{-6} \times 0.0433t^2)$

In Table X. the observed and the calculated cubical expansion by heat, between 0° and 100°, of the above alloys are given. The calculations are based on the assumption that the coefficient of expansion of an alloy is equal to the mean of the coefficients of the component metals (expressed in volumes).

TABLE X.

	Vol. per cent.	Observed vol. at 0°=1, then vol. at 100°=	Calculated vol. at 0°=1, then vol. at 100°=
Sn ₄ Pb	22.28 of Pb	1.007188	1.007225
Pb ₄ Sn	82.09 of Pb	1.008419	1.008128
Cd Pb	58.49 of Pb	1.009138	1.008847
Sn ₄ Zn	87.46 of Sn	1.007184	1.007144
Sn ₆ Zn	91.28 of Sn	1.007058	1.007066
Bi ₄₄ Sn	0.85 of Sn	1.004064	1.003972
Bi Sn ₂	42.81 of Sn	1.005098	1.005207
Bi ₂₄ Pb	1.76 of Pb	1.004086	1.004026
Bi Pb ₂	46.26 of Pb	1.008621	1.006007
Cu+Zn (71 per cent. Cu).	33.85 of Zn	1.005719	1.006328
Au Sn ₂	60.85 of Sn	1.004233	1.005919
Au ₂ Sn ₇	73.14 of Sn	1.004428	1.006223
Ag ₄ Au	19.86 of Au	1.005166	1.005549
Ag Au	49.79 of Au	1.004916	1.005123
Ag Au ₄	79.86 of Au	1.004300	1.004693
Ag+Pt (66.6 per cent. Ag)	19.65 of Pt	1.004568	1.005207
Au+Cu (66.6 per cent. Au)	48.06 of Au	1.004657	1.004716
Ag+Cu (36.1 per cent. Ag)	28.31 of Ag	1.005436	1.00533
Ag+Cu (71.6 per cent. Ag)	71.13 of Ag	1.005713	1.00507.

In Table XI. the equivalents and specific gravities used for the foregoing calculations are given.

TABLE XI.

Metal.	Equivalent.	Specific gravity.
Cadmium	56.0	8.655
Zinc	32.6	7.148
Lead	103.7	11.376
Tin	58.0	7.294
Silver	108.0	10.468
Copper	31.7	8.950
Gold	197.0	19.265
Bismuth	208.0	9.822
Platinum	—	21.400

On comparing the observed with the calculated volumes, we find that they mostly agree together as well as may be expected, considering that the observed values cannot be deemed absolutely correct, and that a difference in the crystalline form will in all probability cause a slight difference in the coefficients of expansion. It is well known that alloys crystallize much more readily, in most cases, than the component metals, and not always in the same form. The difference between the observed and calculated values obtained for the alloy Bi Pb₂ is so great that I thought some error had occurred in making the alloy; I therefore remade it, redetermined the coefficient of expansion (Series No. 42), and obtained the same values. That the gold-tin alloys have lower coefficients of expansion than those of the mean of the component volumes of the metals

forming them is not surprising, as in all probability there exist chemical combinations between the two metals,—just as it may be said *that the specific gravity of an alloy is approximately equal to the mean specific gravities of the volumes of the component metals*, so also from the foregoing we may deduce *that the volume which an alloy will occupy at any temperature between 0° and 100° is approximately equal to the mean of the component volumes of the metals at the same temperature*, or, in other words, *the cubical or linear coefficients of expansion by heat of an alloy between 0° and 100° are approximately equal to the mean cubical or linear coefficients of expansion by heat of the component metals*.

In Table XII. I have given the values from Table X., together with the observed and calculated specific gravities and conducting-powers of the alloys experimented with.

TABLE XII.

	Vol. 0=1, then vol. 100=		Specific gravity.		Conducting-power.	
	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
Sn ₄ Pb . . .	1·007188	1·007225	8·818	8·203	10·57	10·63
Pb Sn . . .	1·008419	1·008129	10·590	10·645	8·28	8·43
Cd Pb . . .	1·009138	1·008847	10·246	10·246	12·61	13·72
Sn ₄ Zn . . .	1·007184	1·007144	13·22	13·45
Sn ₆ Zn . . .	1·007058	1·007066	12·66	12·84
Bi ₄₄ Sn . . .	1·004064	1·003972	9·803	9·801	0·245	1·28
Bi Sn ₂ . . .	1·005098	1·005207	8·772	8·738	3·96	5·59
Bi ₂₄ Pb . . .	1·004086	1·004026	9·845	9·850	0·257	1·30
Bi Pb ₂ . . .	1·008621	1·006007	10·956	10·541	2·09	4·23
Cu + Zn . . .	1·005719	1·006328	21·70	70·20
(33·85 vol. Zn)						
Au Sn ₂ . . .	1·004233	1·005919	11·833	11·978	14·27	35·51
Au ₂ Sn ₇ . . .	1·004428	1·006223	6·00	28·25
Ag ₄ Au . . .	1·005166	1·005549	12·257	12·215	20·91	94·62
Ag Au . . .	1·004916	1·005123	14·870	14·847	14·59	86·52
Ag Au ₄ . . .	1·004300	1·004693	17·540	17·493	20·91	78·38
Ag + Pt . . .	1·004568	1·004693	6·70	83·60
(19·65 vol. Pt)						
Au + Cu . . .	1·004657	1·004716	12·00	83·25
(48·06 vol. Au)						
Ag + Cu . . .	1·005436	1·005233	67·85	95·00
(28·31 vol. Ag)						
Ag + Cu . . .	1·005713	1·005607	63·00	98·20
(73·13 vol. Ag)						

The specific gravities of some of the alloys are not given, as they were not determined with the others. Their values deduced, with the help of the data given in this paper, would not be correct, as for the present research the castings were made more with the idea of producing a perfect surface than an absolute solid; no doubt many of them had

internal cavities owing to the surface being rapidly cooled, and for this reason I have not given the absolute weights of the metals or the alloys.

It is worth remarking that the observed specific gravity of the alloy Bi Pb₂ differs considerably from the calculated one.

From the conclusions just drawn it would appear that the determinations of the expansion by heat of alloys give in general no indication as to their chemical nature, and that this property belongs to that class of physical properties which does not indicate their chemical nature. In the Report on the Chemical Nature of Alloys*, I have shown that, from the determinations of the electric conducting-power of alloys, we may gain an insight into their chemical nature; and basing my calculations on the hypothesis there propounded, I am at present able to deduce the conducting-power of any alloy which may be considered as a solidified solution of the one metal in the other, although it may differ widely from that calculated from the mean conducting-power of the component metals, as shown in Table XII.

It is proper to point out that the coefficients given in Tables IV., V., VIII., and IX., are those calculated from readings of the ordinary mercury-thermometer, the mercury being contained in a tube of glass. Therefore, except at the two values 0° and 100°, the temperatures indicated by the degrees of the thermometer I used will differ—

I. From the temperature corresponding to the same number of degrees of an air-thermometer.

II. From the temperature corresponding to the same number of degrees of a mercurial thermometer, the mercury being contained in a tube of some substance absolutely unaffected as to its volume by heat, could such be found.

The proper corrections can be deduced from the following Table:—

Temperatures indicated by air-thermometer.	Temperatures indicated by ordinary thermometer at same time.
10·0	10·08
20·0	20·14
30·0	30·18
40·0	40·20
50·0	50·20
60·0	60·18
70·0	70·14
80·0	80·10
90·0	90·05
100·0	100·00

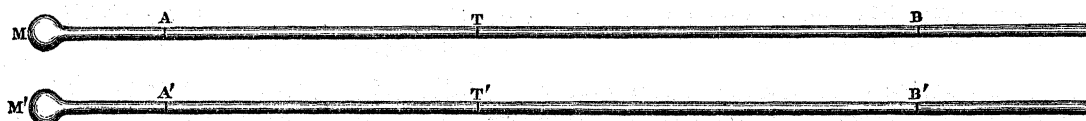
This Table is copied from RECKNAGEL.

REGNAULT found 50° air-thermometer to correspond to 50°·20 by mercurial thermometer.

Professor MILLER has sent me the following investigation of the correction to be applied

* British Association Report, 1863.

to reduce the temperatures indicated by the common mercurial thermometer to those indicated by a thermometer of the second kind mentioned above. I have since found that RECKNAGEL* and POGGENDORFF† have given a similar investigation.



Let M B denote a mercurial thermometer in which A is the freezing-, B the boiling-point, the capacity of the tube between A and B being divided into 100 parts of equal capacity.

Let M' B' denote a thermometer of material absolutely uninfluenced by heat, having exactly the dimensions of A B when at 0° C; A', B' its freezing- and boiling-points, and the capacity of A', B' divided into 100 parts of equal volume.

Let V be the capacity of M A at 0°, 100 K the capacity of A B at 0°. Since the thermometers are exactly alike at 0°, the capacity of M' A' will be V. Let the capacity of A' B' = 100 mV, and, therefore, m the cubic expansion of mercury for 1° in terms of the indications of thermometer M' B'; g the cubic expansion of glass for 1°.

When the thermometers are at the same temperature, let the ends of the columns of mercury stand at T, T', t, t' being the number of degrees between A and T, and A' and T' respectively.

The capacity of M A will now be V(1 + gt'), and that of A T will be Kt (1 + gt'), and the volume of the mercury in M T will be V(1 + mt'). Hence

$$V(1 + mt') = V(1 + gt') + Kt(1 + gt').$$

Therefore

$$V(m - g)t' = Kt(1 + gt').$$

But t and t' are both 100 at the same time. Hence

$$V(m - g)100 = K100(1 + g100).$$

Therefore

$$t' = t \frac{1 + gt'}{1 + g100} = t - gt(100 - t') \text{ very nearly.}$$

But g is very small and t' very nearly equal to t. Hence

$$t' = t - gt(100 - t) \text{ very nearly.}$$

g varies from 0.000023 to 0.000028. Taking the higher value,

$$\text{at } 50^\circ \text{ C., } t - t' = 0.000028 \times 50 \times 50 = 0.07 \text{ C.,}$$

$$\text{at } 25^\circ \text{ C., } t - t' = 0.000028 \times 25 \times 75 = 0.052 \text{ C.}$$

Hence the expansion of mercury is not proportional to its ascent in the tube of a thermometer, and the difference of rates is a perfectly sensible quantity, too large to neglect in many researches. Part of the increased rate of expansion of various bodies at high temperatures is due to this error of the mercurial thermometer, which indicates temperature between 0° and 100° too high as measured by the true expansion of mercury.

* POGGENDORFF's 'Annalen,' vol. cxxiii. p. 115.

† Ibid. vol. xli. p. 372.

REGNAULT frequently used a thermomètre à poids. Let P be the weight of mercury contained in it at 0° C., p the weight of mercury expelled when exposed to the temperature t by ordinary thermometer.

At temperature t by ordinary thermometer the volume of mercury below 0° is $V(1+gt')$, and the volume of the mercury above 0° is $Kt(1+gt')$. Therefore the volume of the whole of the mercury is $V(1+gt') + Kt(1+gt')$.

Hence

$$\frac{p}{P} = \frac{Kt(1+gt')}{V(1+gt') + Kt(1+gt')} = \frac{Kt}{V + Kt}.$$

Therefore

$$\frac{p}{P-p} = \frac{Kt}{V}. \quad \text{When } t=100 \text{ let } p=b.$$

Therefore

$$\frac{b}{P-b} = \frac{K \cdot 100}{V};$$

therefore

$$\frac{t}{100} = \frac{P-b}{P-p} \frac{P}{b}.$$

REGNAULT uses $\frac{P-b}{P-p} \frac{p}{b}$ as the measure of temperature, which is the same as $\frac{t}{100}$, the indication of a common mercurial thermometer uncorrected for the effect produced by the expansion of glass.

A small error is generally made in taking the boiling-point of water, 100° , viz. the influence of latitude is not taken into consideration. Professor MILLER writes to me on this point the following:—"LAPLACE, in the fifth edition of his 'Système du Monde,' states that he regards 100° C. to mean the temperature of water boiling under a pressure equivalent to that of a column of mercury at 0° and 760 millims. in latitude 45° .

"DULONG, and afterwards REGNAULT, assumed 100° to be that of boiling water under normal pressure at Paris. Now the pressure at 760 millims. of mercury at the level of the sea in latitude 45° is equivalent to a column in latitude λ at z metres above the level of the sea, the height of which is $760 \left(1 - 1.32 \frac{z}{r}\right) (1 - 0.0025659 \cos . 2\lambda)$, where r = radius of the earth = 6366198 metres.

"The latitude of Paris ($48^\circ 50' 14''$) is not very different from 45° . REGNAULT's place of observation was about 60 metres above the sea. Hence a column of 760 millims. mercury at the level of the sea exerts the same pressure as a column of 759.75 millims. in REGNAULT's laboratory."

This will only make a difference in the boiling-point of 0.01, a difference which may be neglected in ordinary work, but in normal researches ought to be brought into calculation; as, for example, in MILLER's normal research "On the Construction of the New Standard Pound"*. At Åbo this correction would amount to upwards of $0^\circ.25$.

In the text-books on chemistry and natural philosophy we find the coefficients of

* Philosophical Transactions, 1856, p. 753.

expansion of air given 0.003665 for each degree of the mercury-thermometer, which is not correct; for as air expands regularly between 0° and 100° according to the air-thermometer, a unit of volume of air measured at 0° will not occupy a volume at 20.14 mercury-thermometer equal to

$$1 + 0.003665 \times 20.14 = 1.07380,$$

but

$$1 + 0.003665 \times 20 = 1.07330,$$

showing a difference of 0.05 per cent.

MILLER, in his paper already mentioned, p. 714, says, "REGNAULT found the expansion of air from 0° to 100° under constant pressure equal to 0.36706 of its volume at 0°... The difference between the mercurial and air-thermometers amounts to about 0.2. Hence the expansion of air between 0° and 50°.2 is 0.18353 of its volume at 0°; or between 0° and 50° the ratio of the density of air at 0° to its density at t is $1 + 0.003656 \times t$."

In the present state of science it seems quite wrong that such a want of accordance should exist in our normal instruments, considering that otherwise the errors that exist between the instruments do not amount to 0.1 when properly made. I have had several normal thermometers made for my experiments by Messrs. NEGRETTI and ZAMBRA, and in every case, when compared with my Kew Standard, the agreement between them has been almost perfect.

It would be decidedly a step in the right direction if in future the fact that mercury does not expand regularly between 0° and 100° were taken into account in the construction of all normal thermometers; and if some comparisons between the Kew Standard and the air-thermometer were made, then the readings of thermometers which have been compared with the Kew Standard might be easily corrected.

JOLLY*, in his research into the expansion of water, corrected his thermometers by comparing them with an air-thermometer; unfortunately, however, he does not give a table containing these comparisons; so that his results are obviously not comparable with those obtained with the mercury-thermometer, and therefore at present I am unable to compare his results with those obtained on the same subject by myself. In conclusion, my thanks are due to Dr. M. BEHREND for carrying out the determinations for the Series 1, 2, 13, 20, 21, 24 to 27 and 34, to Mr. BASSETT, who aided me in carrying out the others, and to Mr. R. P. WRIGHT for having undertaken the greater part of the computations.

* Berichte der K. b. Akad. der Wissench. zu München, 1864, p. 141.